King Fahd University of Petroleum and Minerals

College of Computer Science and Engineering

ICS 253 Section 01

Major Exam 1

13 October 2015

Student Name: ______________________________ Student ID: ____________________

Learning Objective	Question	Total Marks	Acquired Marks	Notes
1	1	10		
1&2	2	10		
1	3	10		
1&2	4	10		
2	5	10		
2	6	10		
	Total	**60**		

Notes:

1) Write your Student ID on the top of each paper sheet.
2) This exam contains six different paper sheets **excluding** this cover page.
3) Answer all questions in this exam.
4) Exam duration is 60 minutes.

Question 1

In the island of knights and knaves created by Smullyan, where knights always tell the truth and knaves always lie, you encounter two people, A and B. Determine what A and B are if:

A says: I am a knave or B is a knight

B says: Nothing

Key Solution:

Based on what A said, Let:

p: "I am a knave" ≡ "A is a knave"

q: "B is a knight"

Then we can rewrite the claims as:

A says: $p \vee q$

B says: Nothing

Assume A is a knave, then $p \vee q \equiv False$, which can only be when both p and q are $False$.

If p is false, then this means that A is a knight, which contradicts with our assumption that A is a knave. This means that A cannot be a knave. Thus A is a knight.

Now that A is a knight, this means that $p \vee q \equiv True$, which means that for the statement to be true either p is true or q is true. Since A is a knight, then p is false. Then q must be true. That is, B is a knight

Therefore A is a knight and B is a knight.

Question 2

25. Show that $(p \to r) \vee (q \to r)$ and $(p \wedge q) \to r$ are logically equivalent.

Show, without using the truth table, that $(p \to r) \vee (q \to r)$ and $(p \wedge r) \to r$ are logically equivalent.

Key Solution:

We need to prove that $(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

$LHS \equiv (p \to r) \vee (q \to r)$	Apply $p \to q \equiv \neg p \vee q$ and we get
$\equiv (\neg p \vee r) \vee (\neg q \vee r)$	Series of disjunctions, we remove the brackets

$\equiv \neg p \vee r \vee \neg q \vee r$	Using the commutative law we get
$\equiv \neg p \vee \neg q \vee r \vee r$	Apply the Idempotent law on r and we get
$\equiv \neg p \vee \neg q \vee r$	Apply De Morgan law and we get
$\equiv \neg(p \wedge q) \vee r$	Apply $p \rightarrow q \equiv \neg p \vee q$ and we get
$\equiv (p \wedge q) \rightarrow r \equiv RHS$	

Question 3

Find a common domain for the variables x, y, z, and w for which the statement $\forall x \forall y \forall z \exists w((w \neq x) \wedge (w \neq y) \wedge (w \neq z))$ is true and another common domain for these variables for which it is false.

a) Find a common domain for the variables x, y, z, and w for which the statement $\forall x \forall y \forall z \exists w((w \neq x) \wedge (w \neq y) \wedge (w \neq z))$ is true **and another** common domain for these variables for which it is false.

b) Identify the error or errors in this argument that supposedly shows that if $\exists x P(x) \wedge \exists x Q(x)$ is true then $\exists x(P(x) \wedge Q(x))$ is true:

1.	$\exists x P(x) \wedge \exists x Q(x)$	Premise
2.	$\exists x P(x)$	Simplification from (1)
3.	$P(c)$	Existential instantiation from (2)
4.	$\exists x Q(x)$	Simplification from (1)
5.	$Q(c)$	Existential instantiation from (4)
6.	$P(c) \wedge Q(c)$	Conjunction from (3) and (5)
7.	$\exists x(P(x) \wedge Q(x))$	Existential generalization

Key Solution:

a) Any domain with four or more members makes the statement true;
Any domain with three or fewer members make the statement false.

b) The error occurs in step (5), because we cannot assume, as is being done here, that the c that makes P true is the same as the c that makes Q true.

23. Identify the error or errors in this argument that supposedly shows that if $\exists x P(x) \wedge \exists x Q(x)$ is true then $\exists x(P(x) \wedge Q(x))$ is true.

1.	$\exists x P(x) \vee \exists x Q(x)$	Premise
2.	$\exists x P(x)$	Simplification from (1)
3.	$P(c)$	Existential instantiation from (2)
4.	$\exists x Q(x)$	Simplification from (1)
5.	$Q(c)$	Existential instantiation from (4)
6.	$P(c) \wedge Q(c)$	Conjunction from (3) and (5)
7.	$\exists x(P(x) \wedge Q(x))$	Existential generalization

Question 4

27. Use rules of inference to show that if $\forall x(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x(P(x) \wedge R(x))$ are true, then $\forall x(R(x) \wedge S(x))$ is true.

Use rules of inference to show that if $\forall x(P(x) \rightarrow (Q(x) \wedge S(x))$ and $\forall x(P(x) \wedge R(x))$ are true, then $\forall x(R(x) \wedge S(x))$ is true.

Key Solution

27.

Step	**Reason**
1. $\forall x(P(x) \wedge R(x))$	Premise
2. $P(a) \wedge R(a)$	Universal instantiation from (1)
3. $P(a)$	Simplification from (2)
4. $\forall x(P(x) \rightarrow (Q(x) \wedge S(x)))$	Premise
5. $Q(a) \wedge S(a)$	Universal modus ponens from (3) and (4)
6. $S(a)$	Simplification from (5)
7. $R(a)$	Simplification from (2)
8. $R(a) \wedge S(a)$	Conjunction from (7) and (6)
9. $\forall x(R(x) \wedge S(x))$	Universal generalization from (5)

Question 5

You know that in proof by contradiction we prove that p is true by showing that $\neg p \rightarrow (r \wedge \neg r)$. Prove that $\sqrt{2}$ is irrational by using the proof by contradiction strategy. In your proof, you must indicate which statements constitutes $p, \neg p, r$ and $\neg r$ and then clearly indicate how you reached the contradiction $(r \wedge \neg r)$.

Key Solution

Solution: Let p be the proposition "$\sqrt{2}$ is irrational." To start a proof by contradiction, we suppose that $\neg p$ is true. Note that $\neg p$ is the statement "It is not the case that $\sqrt{2}$ is irrational," which says that $\sqrt{2}$ is rational. We will show that assuming that $\neg p$ is true leads to a contradiction.

If $\sqrt{2}$ is rational, there exist integers a and b with $\sqrt{2} = a/b$, where $b \neq 0$ and a and b have no common factors (so that the fraction a/b is in lowest terms.) (Here, we are using the fact that every rational number can be written in lowest terms.) Because $\sqrt{2} = a/b$, when both sides of this equation are squared, it follows that

$$2 = \frac{a^2}{b^2}.$$

Hence,

$$2b^2 = a^2.$$

By the definition of an even integer it follows that a^2 is even. We next use the fact that if a^2 is even, a must also be even, which follows by Exercise 16. Furthermore, because a is even, by the definition of an even integer, $a = 2c$ for some integer c. Thus,

$$2b^2 = 4c^2.$$

Dividing both sides of this equation by 2 gives

$$b^2 = 2c^2.$$

By the definition of even, this means that b^2 is even. Again using the fact that if the square of an integer is even, then the integer itself must be even, we conclude that b must be even as well.

We have now shown that the assumption of $\neg p$ leads to the equation $\sqrt{2} = a/b$, where a and b have no common factors, but both a and b are even, that is, 2 divides both a and b. Note that the statement that $\sqrt{2} = a/b$, where a and b have no common factors, means, in particular, that 2 does not divide both a and b. Because our assumption of $\neg p$ leads to the contradiction that 2 divides both a and b and 2 does not divide both a and b, $\neg p$ must be false. That is, the statement p, "$\sqrt{2}$ is irrational," is true. We have proved that $\sqrt{2}$ is irrational. ◀

Question 6

17. Suppose that a and b are odd integers with $a \neq b$. Show there is a unique integer c such that $|a - c| = |b - c|$.

Suppose that a and b are odd integers with $a \neq b$. Show that there is a unique integer c such that $|a - c| = |b - c|$

Key Solution

The equation $|a - c| = |b - c|$ is equivalent to the disjunction of two equations:

- $[a - c = b - c] \vee [a - c = -b + c]$

The first of these is equivalent to

$$a - c = b - c$$

$$a = b$$

which contradicts the assumption made in this problem. So the original equation is equivalent to

$$a - c = -b + c$$

$$c = \frac{a + b}{2}$$

Thus there is a unique solution. Furthermore, this c is an integer, because sum of the odd integers a and b is even.